Design and back-testing of a systematic delta-hedging strategy in fx options space

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Abstract

This paper describes design and back-testing of an automated delta-hedging strategy applied to short-dated fx options (specifically – weekly and monthly at-the-money EURUSD straddles).

The results indicate that systematic sale of options that are delta-hedged according to the suggested algorithm generates financial gain for the seller with an attractive Sharpe ratio exceeding 3.0 on after-cost basis (back-testing accounts for volatility bid-offer as well as spot market bid-offer).

For weekly options Sharpe ratio significantly depends on the day of week on which the algorithm systematically sells options: delta-hedging of options sold on Thursdays results in highest Sharpe ratio; delta-hedging of options sold on Fridays results in second-best Sharpe ratio.

The performance of the algorithmic strategy is not correlated with *linear* changes in spot price which is in line with Black-Scholes theory.

The proposed algorithmic strategy has just a few parameters which serves as a natural protection against over-fitting bias. Further fine-tuning of the algorithm requires access to historical data over longer period and/or access to live trading environment.

Keywords: statistical arbitrage, algorithmic trading, delta-hedging, volatility, options, fx

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1 Introduction

The well-known Black-Scholes formula allows one to convert implied volatility of an option into its cash price (Wystup, 2006):

$$C(S,t) = e^{-r_d \tau}\left(fN(d_+) - KN(d_-)\right) \tag{1}$$

$$P(S,t) = -e^{-r_d \tau}\left(fN(-d_+) - KN(-d_-)\right) \tag{2}$$

$$d_\pm = \frac{\ln(f/K) \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{3}$$

$$f = Se^{(r_d - r_f)\tau} \tag{4}$$

where C and P correspond to European vanilla EUR call and EUR put options, S is EURUSD spot price, f - EURUSD forward price, τ - time to expiration, K - option strike, σ - implied volatility of the option, r_d - risk-free rate for domestic currency (USD), and r_f - risk-free rate for foreign currency (EUR).

The sensitivity of option price with respect to change in spot price of the underlying is known as delta risk of the option:

$$\delta_{call} = \frac{\partial C}{\partial S} = e^{-r_f \tau} N(d_+) \tag{5}$$

$$\delta_{put} = \frac{\partial P}{\partial S} = -e^{-r_f \tau} N(-d_+) \tag{6}$$

The sensitivity of delta risk with respect to change in the underlying spot price is known as gamma risk of the option:

$$\gamma_{call} = \gamma_{put} = \frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = e^{-r_f \tau}\frac{n(d_+)}{S\sigma\sqrt{\tau}} \tag{7}$$

The sensitivity of option price with respect to change in time to expiration is known as theta risk of the option:

$$\theta_{call} = \frac{\partial C}{\partial \tau} = -e^{-r_f \tau}\frac{n(d_+)S\sigma}{2\sqrt{\tau}} + \left[r_f S e^{-r_f \tau} N(d_+) - r_d K e^{-r_d \tau} N(d_-)\right] \tag{8}$$

$$\theta_{put} = \frac{\partial P}{\partial \tau} = -e^{-r_f \tau}\frac{n(d_+)S\sigma}{2\sqrt{\tau}} - \left[r_f S e^{-r_f \tau} N(-d_+) - r_d K e^{-r_d \tau} N(-d_-)\right] \tag{9}$$

Change in value of the delta-hedged portfolio (Pf) which consists of a combination of sold options (Op) and the corresponding delta position (δ) can be calculated according to the following formula:

$$Op(S + \Delta S, \tau - \Delta\tau) = Op(S,\tau) + \delta\Delta S + \frac{1}{2}\gamma\left(\Delta S\right)^2 - \theta\Delta\tau \tag{10}$$

$$Pf(S,\tau) = -Op(S,\tau) + \delta \tag{11}$$

$$\Delta Pf(S,\tau) = -\frac{1}{2}\gamma\left(\Delta S\right)^2 + \theta\Delta\tau \tag{12}$$

where θ is sensitivity of Op with respect to changes in time to expiration (τ).

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As one can see from formula (12), the value of the portfolio composed of sold options and respective delta-hedge increases with the mere passage of time (θ-driven component), while changes in the underlying spot price reduce the value (γ-driven component).

Importantly, it is *squared* change in the underlying spot price (we will further refer to this term as hedging error) since last rebalancing that decreases the value of the delta-hedged portfolio, thus just a few changes in the spot price can significantly reduce portfolio value. To avoid this unpleasant scenario a delta-hedger is tempted to rebalance portfolio (i.e. adjust delta component of the portfolio) so that his gamma exposure (hedging error) is minimal. It is not obvious how to implement such rebalancing in practice though.

If the underlying asset price is *volatile* (i.e. if periods of higher price are followed by the periods of lower price and vice versa), then such rebalancing would imply a cost as the seller of the option has to buy the underlying asset when the price of the asset is high and sell it when the price is low, thus suffering the loss.

At expiry of the option, the net financial gain for the seller is summed up of the following terms:

$$pnl = A + B + C \tag{13}$$

where A is the the option premium received from the option buyer (positive amount), B is the cumulative loss (gain) suffered (earned) on delta position (positive or negative amount), and C is the close-out amount (non-positive amount for a straddle)

The same *pnl* amount can be calculated by summing up all changes in the value of the delta-hedged portfolio (see formula 12) from the moment of trade to option expiration.

In the Black-Scholes world the expected cost of *continuous* delta-hedging (terms B and C above) of the vanilla European option from the trade date until option expiration is expected to be equal to the option price, implying no arbitrage for an option seller (or buyer).

In the real world continuous delta-hedging is not feasible due to presence of bid-offer, jumps in the spot price, and non-continuous trading. One notable case when continuous delta-hedging clearly does not work is where an option is close to expiry and the spot price of the underlying asset is close to the option strike: from purely mathematical point of view the gamma risk of an option with just a few moments to expiry and the spot trading around the strike goes to infinity as delta jumps from 0 to 1 (for a vanilla call option).

Motivation

When working on this paper I pursued two goals. My **first** goal was to design a feasible (from practical point of view) systematic delta-hedging strategy that could be used to benchmark potential option trades.

In the past a few basic approaches to delta-hedging were researched (Barles 1998, Zhao 2003, Leland 1985, Sepp 2013, Chen 2010, etc.): some delta-hedging strategies rebalance the portfolio at constant time intervals; some other strategies rebalance the portfolio upon certain deviation in the spot price of the underlying asset. None of the strategies specifically dealt with close-to-expiry options.

The strategy that I focused my research on executes rebalancing whenever the loss accumulated from changes in the underlying spot price exceeds some pre-defined amount (I will further refer to this amount as *threshold*).

My **second** goal was to search options market for potential arbitrage opportunities using designed systematic delta-hedging strategy.

Indeed, it is difficult to imagine that a purely theoretical parameter such as implied volatility can accurately predict future realized volatility which in turn determines the net financial result of sale/purchase of an option (it is worth mentioning that realized volatility itself can be defined and calculated in many different ways).

It is much more natural to assume that market makers price options based on some general statistical properties of the spot market (e.g. recent historical volatility) and, more importantly, actual demand for the options. If option prices set by a market maker turn out to be wrong over some period of time, then she shall adjust the pricing to try to avoid further losses. It seems that it may take the market maker quite a long period of time to realize that her prices are wrong due to: (a) presence of bid-offer that the market maker charges on trades with the customers; (b) potentially offsetting flows (i.e. the market maker can be exposed to very small residual risk if, roughly speaking, the amount of the customers looking to buy is approximately equal to the amount of the customers looking to sell); and (c) lack of widely accepted benchmark against which the performance of option trading could be measured.

It is quite well-known fact that the options typically trade at some premium to expected realized volatility and a systematic sale of options shall be profitable on average (although Sharpe ratio of such strategy shall be rather low). The main focus on my research was therefore on delta-hedging of sold options.

2 Researched market

As mentioned above, presence of transaction costs, as well as non-continuous trading sessions reduce arbitrage opportunities, therefore to maximize my chances for success I focused on the most liquid fx market in the world – EURUSD.

EURUSD options market The over-the-counter EURUSD options market is very liquid: bid-offer spread for 1 week at-the-money options is 0.70 vol, while 1 month at-the-money options are quoted 0.20 vol wide. Unlike exchange-traded options that trade only with pre-defined strikes and expiration days, one can trade an over-the-counter option with the desired maturity and strike at any time.

EURUSD spot market Spot EURUSD market is one of the most liquid markets in the world (certainly most liquid fx market). Bid-offer spread is typically less than 0.0001 (1pip) and trading continues from Sunday evening New York time (early Monday in Australia) to Friday evening New York time – almost 5 full days.

The research was focused on delta-hedging of *sold* at-the-money options because the pricing of these instruments is very transparent (volatility bid-offer tends to be relatively stable over time and end-of-day data can be obtained from various sources such as, for example, Bloomberg).

The research was focused on delta-hedging of *straddles* (a combination of a call and a put with the same at the money strike) because it seemed that systematic delta-hedging of non-linear derivatives with highest gamma exposure should yield the most interesting results.

3 Simplified description of the algorithmic strategy

Systematic delta-hedging strategy researched in this paper is as follows:

1. The algorithm sells an at-the-money EURUSD straddle with 1 week (or 1 month) to expiry with USD notional of 10 million.

2. Typically delta risk of an at-the-money straddle is close to zero, but initial rebalancing with an instant market order may be required ($spot_{prev}$ variable shall contain the rate of the first rebalancing).

3. The algorithm then calculates the gamma-risk ($\gamma_{portfolio}$) of the straddle as a sum of gamma-risk of the call and gamma-risk of the put according to the formula (7) above.

4. The algorithm recalls the formula for changes in value of delta-hedged portfolio (see formula (12)) and calculates deviation in spot that would reduce the value of the portfolio by X (X corresponds to *threshold* parameter mentioned earlier). Importantly, any gain that the algorithm may earn due to mere passage of time (θ-driven component of the formula) is disregarded at this stage. Such change in spot (I will further refer to it as *step*) is calculated according to the following formula:

$$step(X) = min\left(\sqrt{\frac{2X}{\gamma_{portfolio}}}, max\_step\right) \tag{14}$$

   where max_step is an implicit parameter of the algorithm set to 150pips (it imposes a cap on potential *step* values which may be useful when gamma-risk of an option is close to zero, i.e. when an option is deeply in or out of the money).

5. The algorithm calculates new delta risk of the portfolio if spot deviates by *step* from the spot level corresponding to the previous rebalancing: δ_{+step} and δ_{-step}. To make the portfolio delta-neutral at the new spot level, the algorithm will have to buy / sell pre-determined amount of the underlying asset.

6. The algorithm places two stop orders in the market: the top order buys the $(\delta_{+step} - \delta_{prev})$ EUR if price gets to $spot_{prev} + step$ and the bottom order sells $(\delta_{-step} - \delta_{prev})$ EUR if price gets to $spot_{prev} - step$.

7. If one of the stop orders is executed, $spot_{prev}$ is updated and the algorithm goes back to (3).

4 Extra features of the algorithm

Rebalancing at the end of Friday

Generally speaking, when the market opens following the weekend break, the spot price is equally likely to be above and below the Friday's closing price. Given quadratic nature of hedging error (see formula 12) it is advisable to rebalance portfolio exactly at Friday's closing price.

Since trading close to the end of weekly trading session may not be feasible due to evaporating liquidity, to minimize potential exposure to opening gap the algorithm cancels any existing stop orders 15 minutes (an implicit parameter of the algorithm that can be optimized) prior to the end of the trading session and rebalances the portfolio using an instant market order. Having rebalanced the portfolio, the algorithm recalculates stop orders and places them in the market for the remaining 15 minutes (to make sure the algorithm rebalances the portfolio if a large move in spot price occurs until the end of the trading session).

Orders placed at the beginning of trading session on Sunday

Upon the start of the new weekly trading session, the algorithm recalculates the risks of the portfolio using the prevailing spot rate.

If the difference between the prevailing spot rate and the rate of last rebalancing (which occurred not earlier than 15 minutes prior to the end of the trading session on Friday) does not exceed the new *step*, the algorithm recalculates and places regular stop orders in the market (these orders are calculated using previous delta position, but new spot rate).

If, however, the prevailing spot rate is significantly lower or higher than the rate of the last rebalancing (i.e. the market has opened with a gap that exceeds the new *step*), the stop orders are calculated in line with the following rules:

- if $spot_{new} > spot_{prev} + step$, then the bottom order is a regular one (placed at $spot_{new} - step$, and the top order is placed at $spot_{new} + 5pips$;
- if $spot_{new} < spot_{prev} - step$, then the top order is a regular one (placed at $spot_{new} + step$), and the bottom order is placed at $spot_{new} - 5pips$;

where $5pips$ is an implicit parameter of the strategy that can be optimized.

General idea behind the modified rules for orders placement is to try to give the algorithm a chance to monetize potential retracement of the spot rate to the level where the market closed on Friday: if following the calculation of the new stop orders the spot price deviates further from the rate of the previous rebalancing, then the algorithm will accept the loss and rebalance the portfolio. But if the spot rate moves in the opposite direction and the gap between the previous rebalancing rate and the prevailing spot rate reduces, the algorithm will rebalance the portfolio only when the spot market advances by at least new step value.

Further research of behaviour of spot price following large opening gap is required to verify whether the aforementioned procedure is of any value. Lack of detectable retracement of spot price back to Friday's close may imply that upon the start of the new weekly trading session the algorithm simply has to rebalance the portfolio using an instant market order and then follow the regular procedure.

Secondary orders

Along with the orders mentioned above (further I will refer to them as primary orders) the algorithm uses secondary orders that are meant to protect the accumulated value against ultra rapid changes in the spot price.

While primary orders are set at $spot_{prev}+step$ and $spot_{prev}-step$, secondary orders are placed at the following levels:

- secondary top orders:
  - $spot_{prev} + step + step'$;
  - ...
  - $spot_{prev} + step + 4step'$;
- secondary bottom orders:
  - $spot_{prev} - step - step'$,
  - ...
  - $spot_{prev} - step - 4step'$

and $step'$ is calculated according to the following heuristics:

$$step'(X) = max(5pips, step(1.5 \times X) - step(X))$$

where $step(\bullet)$ is the function that returns the size of the $step$ for any given threshold (see formula 14).

Whether secondary orders are indeed required for good performance of the algorithm is a big question: if one has access to high frequency infrastructure and is able to recalculate stop orders before the spot market moves to the levels implied by those orders, then secondary orders will be an unnecessary complication. If, however, one is not certain about the ability of the available infrastructure to timely react to rapid changes in the spot market (significant changes in spot may occur during a fraction of a second), then the algorithm probably should place secondary orders along with the primary ones to make sure the portfolio is automatically rebalanced in "blind zone".

The amount of secondary orders (4) and the formula for their calculation above are implicit parameters of the strategy and can be optimized.

Close-to-expiry regime

An option approaching expiration when spot price is close to strike is extremely difficult to delta-hedge. From the mathematical point of view the delta of a straddle changes rapidly from -100% to +100% ($\gamma \to \infty$) which, if the algorithm had to follow the usual routine for rebalancing, would mean that large portions of underlying shall be transacted at a fraction of a pip.

To avoid this the algorithm switches to special close-to-expiry regime 1 hour prior to rebalancing. First of all, the algorithm cancels existing stop orders and rebalances the portfolio using an instant market order.

From this moment until expiry of the option the Black-Scholes framework is disregarded and new orders will be calculated based on pre-determined piecewise-constant function (see Figure 1).

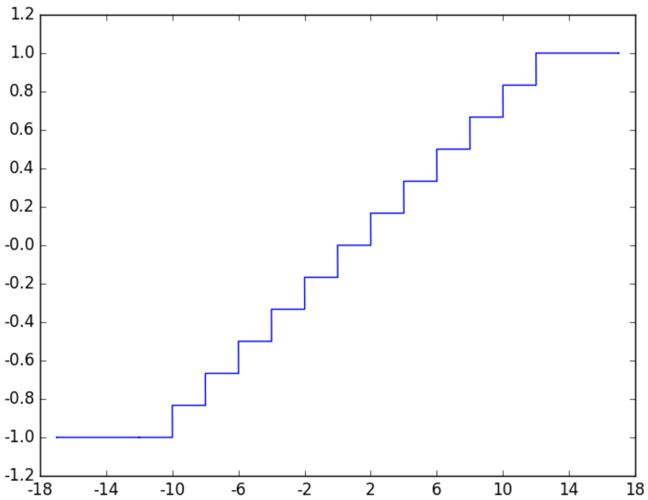


Figure 1: Piece-wise constant function used to calculate delta risk of a short straddle that is close to expiry. Horzontal axis shows distance to strike (in pips); vertical axis shows the amount of delta (1.0 = 100% of delta)

Amount of each stop order will be fixed at 1/6 of max delta amount, and step for the orders will be fixed at 2 pips.

Despite seemingly simple idea, it has to be implemented in actual code with care: when the close-to-expiry regime is switched on, the orders will have to be adjusted for the existing delta position of the portfolio. It could be the case, for example, that the portfolio has accumulated full delta position and the spot is trading just 1 pip below the strike – the algorithm will place no downside orders in the market (since the downside delta position has already been accumulated), but the first upside order will be exactly at strike of the option with the notional equal to the full delta position (previously accumulated position will have to be unwound if the spot price moves higher).

The size of the step and the notional amount of the order are implicit parameters of the algorithm that can be optimized.

It looks like the maximum notional of the option that can be delta-hedged using the suggested strategy is limited specifically by this regime: the underlying market should be liquid enough to allow transacting material notional amounts at minimal (or no) slippage.

Forced delta-hedging upon approaching expiry

One minute prior to expiration (an implicit parameter that could be optimized) of the option the algorithm cancels all previously placed and unexercised orders and uses an instant market order to increase accumulated hedge-delta position to +100% or -100% depending on whether the spot is above or below the strike.

Adjustment of the distant orders on the expiration day

The delta and gamma risks calculated by the algorithm at some point in time change as the option is approaching expiry even if spot rate stays unchanged. If the option is far from expiration, it is safe to assume that delta and gamma do not change significantly until the next rebalancing takes place (and new orders will be calculated using new market spot rate and new time to expiry, thus adjusting the risks for the time that passed since the previous rebalancing).

If the option is about to expire, the sensitivity of delta and gamma with respect to time to expiry significantly increases, and the algorithm would need to recalculate the risks (and replace the existing orders with the new ones) even between rebalancing acts.

One specific case where the issue of increased sensitivity of risks with respect to time is obvious is where an option is deeply in the money (spot is above strike) and has a few hours to expiry: gamma risk of the option will be small, which implies that *step* will be rather high (see formula 14 above). Obviously the gamma risk will increase if spot rate starts dropping towards strike, but since the strategy does not take into account market information until the next rebalancing takes place and the orders are calculated based on $\gamma_{portfolio}$ observed at single point in time, it could well be the case that the first stop order will be formally placed below the strike, which would lead to inefficient delta-hedging and significant losses.

The most simple (although not ideal) solution that was implemented to address the issue is to limit *step* to the distance the between prevailing spot rate and the strike – at least the algorithm will ensure that negative (positive) delta amount will not be transferred to the below(above)-strike area.

This special regime limiting *step* for bottom or top orders (depending on whether the spot is above or below the strike) switches on only if: (1) the time to expiration is less than 1 day; (2) *step* calculated according to formula 14 is equal to max_step; and (3) the distance to strike from the current level of spot is less than max_step.

5 A few important assumptions

Constant volatility and no smile

The algorithm assumes that the volatility remains constant from the moment of trade until option expiration; presence of potential volatility smile is also ignored. Although it may be regarded as a drawback of the algorithm, it is not obvious that non-constant volatility or volatility smile could materially improve performance of the algorithm without affecting its robustness.

Indeed, if the observed results are correct, then implied volatility is not an accurate predictor of the future realized volatility, and one may find it questionable that one should rely on changes in the incorrect predictor or its higher-order complications (smile).

Another important factor to consider is that volatility/smile tick data is not readily available.

Constant interest rate differential

The algorithm does not have access to actual historical USD and EUR rates data: it assumes that USD risk free rate is constant and equal to 0.15% per annum. EUR rate is implied from the EURUSD forward points which are observed in the market. EUR and USD interest rates are assumed to be constant until expiration of the option.

Partial execution of the order-execution

The algorithm assumes that 100% of notional of each stop order is executed.

Slippage

Stop orders may experience some slippage upon execution – actual transacted rate may be different from the rate set by the order parameters. Slippage may materially harm performance and is extremely difficult to research/model – one would need highly specific data to assess the potential impact.

The best publicly available information that I could find is slippage statistics published by broker Saxo bank at `http://www.saxobank.com/prices/forex/order-execution#historical`; the data is summarized in Table 1

Table 1: Historical slippage statistics for EURUSD stop orders. The last column backsolves for fixed slippage that would result in the observed average slippage across all orders.

Period	Total number of stop orders	Number of orders seeing slippage	Percentage of orders filled with no slippage	Average slippage, pips	Indicative slippage, pips
2016 Q1	35934	616	98.3%	0.1	5.8
2015 Q4	41885	1856	95.6%	0.4	9.0
2015 Q3	47974	2561	94.7%	0.1	1.9
2015 Q2	85194	4517	94.7%	0.1	1.9
2015 Q1	83271	4610	94.5%	0.0	0.0
2014 Q4	36585	1531	95.8%	0.0	0.0
2014 Q3	38339	1113	97.1%	0.1	3.4
2014 Q2	41023	1116	97.3%	0.0	0.0
2014 Q1	50989	968	98.1%	0.1	5.3
2013 Q4	59443	1648	97.2%	0.1	3.6
2013 Q3	60866	2133	96.5%	0.1	2.9
2013 Q2	78384	1751	97.8%	0.0	0.0
2013 Q1	93576	2296	97.5%	0.0	0.0
2012 Q4	69334	906	98.7%	0.0	0.0
2012 Q3	95858	2747	97.1%	0.1	3.5
2012 Q2	99095	2203	97.8%	0.0	0.0
2012 Q1	127224	858	99.3%	0.0	0.0
2011 Q4	116833	1615	98.6%	0.1	7.2
2011 Q3	114423	1291	98.9%	0.0	0.0
2011 H1	234619	17647	92.5%	0.2	2.7

The vast majority of all orders was executed with no slippage and average slippage across the whole set of orders is equal to 0.1-0.2 pips. Unfortunately the data about distribution of slippage is not available.

To account for the slippage in my back-testing procedure I assume the following rules for order filling:

- If the market quote deviates from the order rate by less than 10pips, the order is filled at order rate.
- If the market quote deviates from the order rate by more than 10pips, but by less than 20pips, the order is filled at the middle between the order rate and the market rate.
- If the market quote deviates from the order rate by more than 20pips, the order is filled at market rate.

6 Data

To back-test the aforementioned systematic delta-hedging strategy one would need historical dynamics of EURUSD spot price as well as historical volatility data for weekly and monthly EURUSD at-the-money straddles.

EURUSD tick data A few tick datasets can be found in the Internet, I used the data provided by Dukascopy (`http://www.dukascopy.com`). Due to some odd features in the data I disregarded ticks before 01-Jan-2011, thus my dataset was limited to 5 full years (2011-2015). The data contains the information about bid-offer spreads as well as volume for each individual quote. A few features of the tick data are shown in Table 2.

Table 2: Key features of historical EURUSD tick data

year	average bid-offer, pips	average offer size, EUR mm	average bid size, EUR mm	total number of ticks
2011	0.88	2.29	2.30	25'793'802
2012	0.82	2.28	2.27	23'415'406
2013	0.39	2.77	2.76	18'623'653
2014	0.28	2.65	2.60	17'004'494
2015	0.32	2.11	2.01	24'232'002

Volatility data Intraday volatility data is not readily available, but end-of-day at-the-money mid-market volatility could be loaded, for example, from Bloomberg. Importantly, using end-of-day volatility one would be able to price an option that could be traded (for back-testing purposes) at the end of the day only.

Since trading options at the end of the trading day is of little practical use, I perform back-testing for two subsets of options: the first subset contains options that were systematically sold at 10:00 New York time (and priced using end-of-day (17:00 New York time) volatility observation); the second subset contains options that were systematically sold at 16:30 New York time (and priced using the same end-of-day (17:00 New York time) volatility observation). Please note that in the first subset the weekly options have exactly 7 days (168 hours) to expiration, while in the second subset – only 6 days, 17 hours and 30 minutes (161.5 hours).

The discovered effects present in both subsets implying that the time gap between the moment of sale and the moment the volatility observation was collected that presents in the first subset does not materially impact the validity of results. I use only one subset for monthly options (options are traded at 10:00 New York time and priced using volatility observation collected at 17:00 New York time on the same day) I do not expect the existing time gap to materially influence the results for longer-dated options.

Volatility bid-offer was assumed to be 0.80 for weekly options and 0.30 for monthly options, which is somewhat higher than spreads typically charged by market-makers.

7 Results

Back-testing of the delta-hedging algorithm applied to weekly straddles

Realized annual Sharpe ratio (calculated as the average daily return of the algorithmic strategy divided by the standard deviation of daily returns and multiplied by the scaling factor of $\sqrt{252}$) is shown in Tables 3 and 7.

As mentioned earlier, the algorithm was tested using two subsets of weekly straddles. **Subset1** comprises results of delta-hedging of options sold at 10.00 New York time. Since volatility observation used to price these options is collected at 17.00 New York time on the same day, a potential forward-looking bias may jeopardize the validity of the back-testing results. **Subset2** comprises results of delta-hedging of options sold at 16.30 New York time: time gap between the moment of sale and the moment volatility observation is collected is small and can be ignored. Options in this subset are sold at the end of the trading day, therefore time to expiration should be adjusted downwards. It is not clear how market makers would quote options from subset2 in practice.

Importantly, since the key findings are observed in both subsets, the aforementioned time gap was not likely to have material impact on back-testing results.

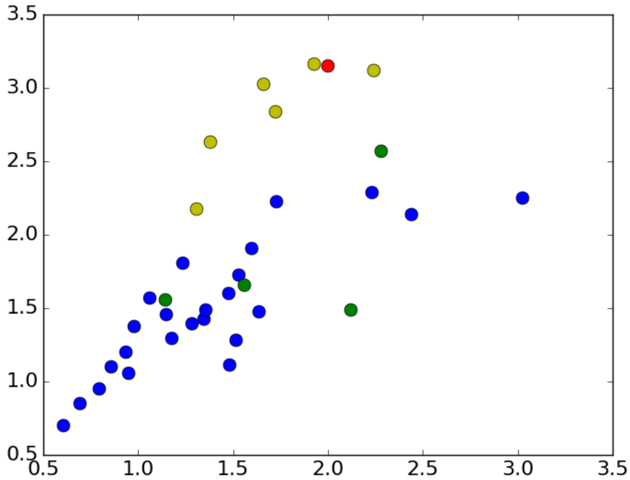
The most notable observation is that the highest Sharpe ratio is realized for options sold on Thursdays, and the second-highest Sharpe ratio is realized for options sold on Fridays – the effect was found for options in both subsets and for each of the back-tested years (2011-2015).

The delta-hedging strategy was back-tested for a range of *threshold* parameters: 100, 500, 1000, 2500, 5000, 7500 and 10000. To select optimal *threshold* and avoid over-fitting one would need to perform out-of-sample testing which was not feasible in this research given relatively scarce available data.

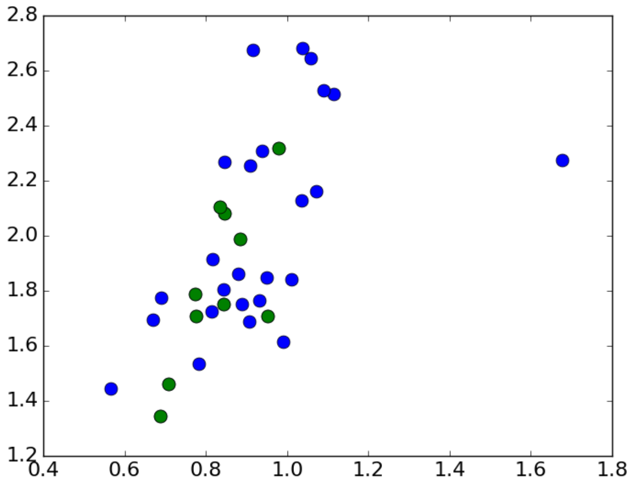
It seems that *threshold* of 2500 may be close to optimal based on the following observations: (1) the average of the respective Sharpe ratios is high; (2) the $\frac{average}{st.deviation}$ ratio of five (2011-2015) annual ratios is also high (please also refer to Figure 2a). Since my back-testing assumes that the value of the portfolio is unchanged between two rebalancing acts, Sharpe ratio for large *threshold* could be somewhat inflated as large threshold may result in no rebalancing acts during some of the days. Based on that I would somewhat discount the results for *threshold* parameters of 7500 and 10000.

To add credibility to the observed results we shall perform a simple verifica-

Figure 2: Graphs demonstrate performance of delta-hedging strategy applied to weekly and monthly options with different parameters. Vertical axis shows Sharpe ratio averaged over 5 years (2011-2015). Horizontal axis shows the $\frac{average}{st.deviation}$ ratio of annual Sharpe ratios observed over 5 years.



(a) Weekly straddles (subset1). Red dot corresponds to options sold on Thursdays and *threshold* of 2500 (this combination seems to be most attractive for an investor). Yellow dots correspond to options sold on Thursdays; green dots correspond to *threshold* of 2500; blue dots correspond to all other combinations.



(b) Monthly straddles. Green dots correspond to *threshold* of 100 and 500; blue dots correspond to other *threshold* parameters - clearly optimal parameter should exceed 500.

Table 3: Realized Sharpe ratio for weekly straddles

year	threshold	subset1 (10:00)					subset2 (16:30)				
		Mon	Tue	Wed	Thu	Fri	Mon	Tue	Wed	Thu	Fri
average Sharpe	100	0.851	0.699	0.951	2.173	1.488	0.886	0.980	1.543	2.610	1.641
	500	1.396	1.295	1.569	3.023	1.909	1.428	1.933	2.309	3.389	2.220
	1000	1.724	1.426	1.807	2.837	2.223	1.495	1.850	2.427	3.508	2.385
	2500	1.485	1.656	1.557	3.148	2.570	1.787	1.831	2.295	3.053	2.670
	5000	1.375	1.597	1.054	3.164	2.285	1.753	1.491	2.070	3.611	2.815
	7500	1.114	1.474	1.200	3.118	2.136	1.601	1.065	2.049	3.279	2.221
	10000	1.284	1.103	1.457	2.629	2.252	1.650	1.862	2.011	2.966	2.041
ratio of average to standard deviation of Sharpe ratios	100	0.695	0.607	0.795	1.308	1.358	0.822	0.723	1.497	1.482	1.592
	500	1.282	1.175	1.060	1.659	1.596	1.334	1.371	1.923	2.021	2.049
	1000	1.531	1.349	1.234	1.726	1.729	1.652	1.380	2.048	1.911	2.179
	2500	2.120	1.559	1.144	2.000	2.281	1.795	1.752	2.310	2.036	2.170
	5000	0.980	1.479	0.951	1.929	2.234	2.256	1.449	2.361	2.282	2.769
	7500	1.481	1.639	0.936	2.244	2.439	1.418	0.972	2.328	2.102	1.864
	10000	1.514	0.858	1.146	1.381	3.024	1.659	1.649	1.662	1.336	2.346

tion test: we will calculate realized correlation between the net financial results of individual cycles (one cycle corresponds to delta-hedging of one sold option) and (1) changes in spot price realized over the respective cycle, (2) number of ticks representing evolution of spot dynamics over the respective cycle.

Net financial gain or loss of the delta-hedging strategy is expected to be immune with respect to *linear* changes in the underlying spot price (see formula 12) – the correlation realized over back-testing period shall be zero. Performance of the strategy shall not depend on the number of observations (ticks) available for each given cycle: the vast majority of tick data corresponds to small changes (less than 1pip) in the underlying spot price, while the pnl record is impacted only when significant change in the spot price occurs. Realized correlation values (Pearson correlation) as well as respective p-values are shown in Table 4.

Back-testing of delta-hedging algorithm applied to monthly straddles

Annual Sharpe ratio (defined as above: the average daily return of the algorithmic strategy divided by the standard deviation of daily returns adjusted by the scaling factor of $\sqrt{252}$) is shown in the Tables 5 and 8. Labels st1, st2, st3, st4 and st5 are arbitrary: if an option from st1 subset is sold on day T, then the next option from st2 subset would be sold on day T+4, the next option from st3 would be sold on T+8, and so on.

Back-testing results for monthly options are more volatile which is the result of significantly smaller number of available cycles (1 monthly cycle corresponds to 4 weekly cycles). Based on the available back-testing results it is difficult to comment on optimal *threshold*, although it is clear that the optimal range for the parameter lies above 500 (see Figure 2b).

As we did before, we perform a simple verification test: we calculate realized correlation between the net financial result of an individual cycle and: (1) change in spot price realized over the cycle, (2) number of ticks. Realized correlation values (Pearson correlation) as well as respective p-values are shown in Table 6.

8 Discussion

In this paper we designed and back-tested an algorithmic delta-hedging strategy that could be automated to risk-manage portfolio of sold options.

Table 4: Verification of back-testing of the delta-hedging strategy applied to weekly straddles

threshold	subset1 (10:00) Mon	Tue	Wed	Thu	Fri	subset2 (16:30) Mon	Tue	Wed	Thu	Fri
	correlation between net financial result and **spot change**									
100	0.041	0.006	-0.008	-0.007	0.079	0.037	-0.009	-0.029	0.032	0.063
500	0.040	0.003	0.002	0.023	0.084	0.021	-0.016	-0.026	0.046	0.057
1000	0.020	0.003	0.022	0.004	0.089	0.025	-0.005	-0.014	0.032	0.066
2500	0.015	-0.023	0.013	-0.005	0.032	-0.029	-0.026	-0.029	0.013	0.058
5000	-0.030	-0.044	0.011	-0.055	0.037	-0.014	-0.047	0.009	-0.000	0.011
7500	-0.028	-0.062	0.005	-0.012	-0.043	-0.039	-0.043	0.013	-0.041	-0.062
10000	-0.033	-0.027	0.009	-0.083	-0.016	-0.021	-0.110	-0.037	-0.080	-0.006
	p-value (0.1=10%)									
100	0.529	0.924	0.899	0.911	0.210	0.573	0.891	0.645	0.611	0.319
500	0.538	0.965	0.972	0.711	0.179	0.747	0.793	0.670	0.466	0.365
1000	0.760	0.957	0.728	0.950	0.156	0.703	0.932	0.821	0.608	0.291
2500	0.819	0.711	0.837	0.939	0.609	0.661	0.671	0.646	0.842	0.358
5000	0.651	0.479	0.863	0.385	0.552	0.829	0.447	0.884	0.997	0.863
7500	0.669	0.321	0.938	0.844	0.492	0.549	0.485	0.835	0.516	0.325
10000	0.616	0.665	0.878	0.184	0.800	0.756	0.075	0.556	0.200	0.923
	correlation between net financial result and **number of ticks**									
100	0.036	-0.010	-0.100	-0.037	0.045	0.024	0.008	-0.077	-0.021	0.050
500	-0.001	-0.061	-0.099	-0.061	0.029	0.001	-0.036	-0.105	-0.060	0.012
1000	0.004	-0.062	-0.120	-0.059	0.037	-0.032	-0.043	-0.111	-0.033	0.032
2500	0.011	-0.055	-0.097	-0.072	0.035	-0.014	-0.015	-0.139	-0.024	-0.013
5000	0.045	-0.053	-0.125	-0.064	0.031	0.020	-0.024	-0.095	-0.044	0.028
7500	-0.029	-0.031	-0.133	-0.094	0.004	0.053	-0.073	-0.099	-0.050	0.016
10000	-0.007	-0.053	-0.139	-0.058	-0.019	0.049	-0.030	-0.097	-0.021	-0.056
	p-value (0.1=10%)									
100	0.583	0.870	0.107	0.558	0.475	0.715	0.894	0.211	0.740	0.426
500	0.993	0.327	0.109	0.329	0.639	0.983	0.557	0.090	0.334	0.846
1000	0.950	0.320	0.052	0.346	0.551	0.626	0.493	0.074	0.600	0.616
2500	0.862	0.373	0.120	0.248	0.577	0.835	0.803	0.024	0.701	0.843
5000	0.495	0.395	0.044	0.306	0.622	0.766	0.704	0.125	0.483	0.653
7500	0.655	0.615	0.031	0.134	0.945	0.416	0.239	0.112	0.424	0.802
10000	0.915	0.396	0.025	0.356	0.761	0.459	0.629	0.119	0.740	0.375

Table 5: Realized Sharpe ratio for 5 subsets of monthly straddles

threshold	st1	st2	st3	st4	st5
	average				
100	1.345	1.460	1.987	1.707	1.786
500	1.708	1.749	2.317	2.080	2.104
1000	1.724	1.805	2.643	2.307	2.268
2500	1.840	1.765	2.513	1.694	2.252
5000	1.613	1.846	2.161	1.750	1.915
7500	1.535	1.774	2.679	2.528	2.128
10000	1.860	1.445	2.674	1.687	2.272
	ratio of average to standard deviation				
100	0.689	0.710	0.885	0.776	0.774
500	0.953	0.844	0.981	0.846	0.836
1000	0.815	0.844	1.059	0.939	0.847
2500	1.011	0.933	1.116	0.670	0.911
5000	0.992	0.952	1.074	0.890	0.817
7500	0.784	0.691	1.039	1.092	1.037
10000	0.881	0.567	0.918	0.909	1.678

Table 6: Verification of back-testing of the delta-hedging strategy applied to monthly straddles

threshold	st1	st2	st3	st4	st5	st1	st2	st3	st4	st5
	correlation vs. **spot change**					correlation vs. **number of ticks**				
100	-0.154	-0.059	0.061	0.041	0.017	-0.004	0.040	-0.051	-0.066	-0.037
500	-0.168	-0.048	0.053	0.043	0.021	-0.017	0.054	-0.064	-0.076	-0.089
1000	-0.141	-0.054	0.045	0.035	-0.038	0.014	0.089	-0.061	-0.106	-0.028
2500	-0.198	-0.049	0.067	0.051	-0.084	-0.022	0.042	-0.096	-0.068	-0.056
5000	-0.175	-0.059	0.051	0.024	-0.063	-0.011	0.041	-0.195	-0.142	-0.126
7500	-0.152	-0.025	0.025	-0.041	-0.110	0.038	0.150	-0.163	-0.161	-0.187
10000	-0.169	-0.035	0.027	-0.030	-0.061	0.049	-0.001	-0.080	-0.193	-0.167
	p-value (0.1=10%)					p-value (0.1=10%)				
100	0.228	0.644	0.638	0.750	0.893	0.972	0.754	0.691	0.610	0.775
500	0.189	0.708	0.684	0.738	0.868	0.897	0.674	0.623	0.551	0.490
1000	0.269	0.673	0.729	0.788	0.769	0.915	0.490	0.637	0.408	0.826
2500	0.121	0.702	0.608	0.689	0.513	0.864	0.746	0.459	0.597	0.666
5000	0.171	0.645	0.692	0.853	0.623	0.931	0.751	0.129	0.265	0.326
7500	0.235	0.844	0.845	0.750	0.391	0.768	0.242	0.205	0.207	0.143
10000	0.186	0.786	0.837	0.814	0.637	0.702	0.997	0.536	0.129	0.192

One of the advantages of the proposed strategy is that it automatically adjusts rebalancing frequency depending on gamma-risk of the portfolio: algorithm places stop orders (above and below the previous rebalancing level) in the market so that the delta-hedged portfolio would suffer not more than a fixed pre-determined loss upon execution of any of the orders (the loss will be offset by potential gain that the algorithm may earn because of passage of time).

Historical data suggests that slippage, that could affect execution rate on stop orders, is not material at least for small notional of the hedged options and liquid underlying market.

Specific close-to-expiry regime was introduced to deal with at-the-money options that have less than 1 hour to expiration.

The algorithmic delta-hedging strategy was back-tested using monthly and weekly at-the-money EURUSD straddles. Transaction costs (spot price bid-offer: less than 1pip; volatility bid-offer: 0.8-wide for weekly straddles and 0.3-wide for monthly straddles) were taken into account. Available data covered five calendar years from 2011 to 2015.

The results show that the proposed delta-hedging strategy allows for effective extraction of volatility premium from the options market: systematic delta-hedging of sold options generated net financial gain for the seller with attractive Sharpe ratio (above 3.0 for weekly straddles, and above 1.7 for monthly straddles).

One of the most non-trivial results of the research was that delta-hedging of weekly straddles sold on Thursdays would generate significantly higher Sharpe ratio compared to straddles sold on Mondays, Tuesdays and Wednesdays (delta-hedging of the options sold on Fridays generates second-highest Sharpe ratio).

Importantly, the performance of the strategy, back-tested with a range of *threshold* values, demonstrates that the parameter could be calibrated to some optimum (back-testing of weekly straddles suggest that the optimal *threshold* could be around 2500; back-testing of monthly straddles implies that the optimal *threshold* shall exceed 500).

Simple verification test was performed to make the results more reliable:

correlation between the net financial result realized over an investment cycle (delta-hedging of one option from the moment of sale until its expiration constitutes one investment cycle) and: (1) change in spot price realized over the same investment cycle; and (2) number of ticks in the dataset for the investment cycle turned out to be non-significant from statistical point of view (as expected).

The same strategy with minor adjustments (stop orders will have to be replaced with take-profit orders) could be applied to the portfolio of long options, and the volatility bid-offer shall help market-makers to alleviate negative drift in their pnl record due to presence of volatility premium.

9 Bibliography

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10 Appendix

Table 7: Realized Sharpe ratio for weekly straddles (full table)

year	threshold	**subset1 (10:00)**					**subset2 (16:30)**				
		Mon	Tue	Wed	Thu	Fri	Mon	Tue	Wed	Thu	Fri
	100	2.876	2.436	2.611	4.832	2.922	2.666	3.147	2.818	5.167	2.966
	500	3.176	2.912	3.309	5.525	3.337	3.016	3.807	3.483	5.475	3.448
	1000	3.439	2.580	3.588	5.237	3.826	2.741	3.677	3.682	6.074	4.040
2011	2500	2.594	2.892	3.416	4.796	3.601	3.250	3.350	2.891	4.829	3.556
	5000	2.748	3.147	2.553	5.122	3.704	3.059	2.779	3.351	5.798	4.069
	7500	2.333	2.917	2.998	4.259	3.284	3.295	2.407	3.193	4.979	4.056
	10000	2.021	3.171	2.808	5.450	2.873	3.174	3.367	3.238	5.719	2.532
	100	0.934	0.867	1.502	2.006	2.141	1.074	0.793	1.795	2.636	1.759
	500	1.551	1.570	2.853	3.874	3.057	2.046	2.196	3.232	4.048	3.060
	1000	2.271	2.433	2.950	3.346	3.340	2.182	2.476	3.236	4.101	2.845
2012	2500	1.758	2.546	2.466	4.317	3.752	2.366	2.130	3.569	4.148	4.162
	5000	2.994	2.082	1.498	3.625	2.939	1.662	2.139	2.012	3.920	2.978
	7500	0.683	1.685	1.612	3.492	2.033	2.203	0.824	2.277	3.992	2.475
	10000	2.200	1.350	1.448	2.787	2.563	1.311	1.196	1.456	3.757	1.562
	100	-0.371	0.744	0.385	0.704	0.065	0.015	1.031	1.028	1.235	0.071
	500	0.634	1.151	0.804	1.314	0.852	0.521	2.222	1.835	2.116	0.791
	1000	1.255	1.223	1.330	1.410	1.008	0.851	1.740	1.960	2.325	1.291
2013	2500	1.119	1.406	0.899	1.923	1.753	1.301	1.853	1.877	1.979	0.998
	5000	0.276	1.235	0.654	1.343	1.293	1.492	1.479	2.326	2.066	1.311
	7500	1.305	0.720	0.469	2.243	1.070	0.914	1.861	2.221	2.358	0.966
	10000	1.232	0.043	0.866	1.377	1.597	2.078	2.007	2.401	1.985	0.827
	100	0.318	0.136	0.827	2.459	1.346	0.136	0.492	1.983	3.284	1.641
	500	1.188	0.978	1.135	3.203	1.367	0.854	1.535	2.498	4.023	1.558
	1000	0.819	0.735	1.242	3.009	1.779	0.927	1.272	2.600	3.783	1.627
2014	2500	1.043	1.150	0.989	3.590	2.572	1.161	1.284	2.181	3.122	2.303
	5000	0.856	1.257	1.032	4.135	1.449	1.576	0.926	1.696	4.206	2.492
	7500	0.824	1.287	1.337	4.427	2.694	1.026	0.628	1.783	4.014	1.402
	10000	0.804	0.901	2.497	3.084	2.909	0.703	2.344	2.770	3.590	2.218
	100	0.500	-0.690	-0.572	0.866	0.965	0.539	-0.562	0.092	0.730	1.770
	500	0.433	-0.136	-0.254	1.197	0.931	0.703	-0.097	0.499	1.283	2.241
	1000	0.834	0.159	-0.077	1.185	1.164	0.776	0.086	0.656	1.258	2.121
2015	2500	0.911	0.287	0.016	1.115	1.173	0.859	0.538	0.959	1.186	2.330
	5000	-0.001	0.263	-0.466	1.596	2.041	0.977	0.134	0.964	2.064	3.225
	7500	0.427	0.763	-0.418	1.168	1.597	0.569	-0.393	0.770	1.054	2.205
	10000	0.162	0.050	-0.334	0.449	1.317	0.985	0.398	0.190	-0.221	3.066
	100	0.851	0.699	0.951	2.173	1.488	0.886	0.980	1.543	2.610	1.641
	500	1.396	1.295	1.569	3.023	1.909	1.428	1.933	2.309	3.389	2.220
	1000	1.724	1.426	1.807	2.837	2.223	1.495	1.850	2.427	3.508	2.385
average Sharpe	2500	1.485	1.656	1.557	3.148	2.570	1.787	1.831	2.295	3.053	2.670
	5000	1.375	1.597	1.054	3.164	2.285	1.753	1.491	2.070	3.611	2.815
	7500	1.114	1.474	1.200	3.118	2.136	1.601	1.065	2.049	3.279	2.221
	10000	1.284	1.103	1.457	2.629	2.252	1.650	1.862	2.011	2.966	2.041
ratio of average to standard deviation of Sharpe ratios	100	0.695	0.607	0.795	1.308	1.358	0.822	0.723	1.497	1.482	1.592
	500	1.282	1.175	1.060	1.659	1.596	1.334	1.371	1.923	2.021	2.049
	1000	1.531	1.349	1.234	1.726	1.729	1.652	1.380	2.048	1.911	2.179
	2500	2.120	1.559	1.144	2.000	2.281	1.795	1.752	2.310	2.036	2.170
	5000	0.980	1.479	0.951	1.929	2.234	2.256	1.449	2.361	2.282	2.769
	7500	1.481	1.639	0.936	2.244	2.439	1.418	0.972	2.328	2.102	1.864
	10000	1.514	0.858	1.146	1.381	3.024	1.659	1.649	1.662	1.336	2.346

Table 8: Realized Sharpe ratio for 5 subsets of monthly straddles (full table)

threshold	st1	st2	st3	st4	st5
			2011		
100	1.386	3.611	3.725	2.912	2.642
500	1.346	3.666	3.813	3.030	2.775
1000	1.508	3.771	4.205	3.393	3.214
2500	1.684	4.077	3.521	3.161	3.126
5000	2.347	4.384	3.492	2.492	2.813
7500	1.514	5.105	2.835	3.795	2.755
10000	1.863	4.161	4.422	2.931	1.917
			2012		
100	4.466	3.593	4.618	4.865	5.251
500	4.610	4.152	5.296	5.744	6.008
1000	5.091	4.401	5.778	5.949	6.359
2500	4.717	3.393	5.193	5.404	6.127
5000	3.851	3.106	3.906	4.800	5.259
7500	4.734	3.346	5.420	5.636	5.053
10000	5.352	3.574	5.933	4.085	4.471
			2013		
100	1.139	-0.621	0.494	0.637	0.299
500	1.696	-0.209	0.856	1.262	0.920
1000	2.002	0.181	1.353	1.006	0.976
2500	2.103	0.819	1.271	0.379	1.269
5000	1.798	1.059	1.183	1.089	2.004
7500	1.505	-0.939	1.236	1.520	2.341
10000	1.667	-0.573	0.683	1.167	2.565
			2014		
100	0.613	1.055	1.933	0.880	1.450
500	1.191	1.309	2.311	1.184	1.535
1000	0.551	0.939	2.547	1.623	1.480
2500	0.815	0.958	3.235	0.244	0.902
5000	0.092	1.306	3.142	0.557	0.295
7500	0.225	1.917	4.772	2.169	0.869
10000	0.463	1.767	3.588	0.940	1.234
			2015		
100	-0.878	-0.338	-0.836	-0.759	-0.714
500	-0.301	-0.173	-0.691	-0.818	-0.716
1000	-0.530	-0.267	-0.669	-0.436	-0.690
2500	-0.117	-0.421	-0.655	-0.719	-0.166
5000	-0.024	-0.626	-0.917	-0.188	-0.798
7500	-0.305	-0.560	-0.870	-0.479	-0.376
10000	-0.045	-1.704	-1.258	-0.689	1.174
			average		
100	1.345	1.460	1.987	1.707	1.786
500	1.708	1.749	2.317	2.080	2.104
1000	1.724	1.805	2.643	2.307	2.268
2500	1.840	1.765	2.513	1.694	2.252
5000	1.613	1.846	2.161	1.750	1.915
7500	1.535	1.774	2.679	2.528	2.128
10000	1.860	1.445	2.674	1.687	2.272
	ratio of average to standard deviation				
100	0.689	0.710	0.885	0.776	0.774
500	0.953	0.844	0.981	0.846	0.836
1000	0.815	0.844	1.059	0.939	0.847
2500	1.011	0.933	1.116	0.670	0.911
5000	0.992	0.952	1.074	0.890	0.817
7500	0.784	0.691	1.039	1.092	1.037
10000	0.881	0.567	0.918	0.909	1.678

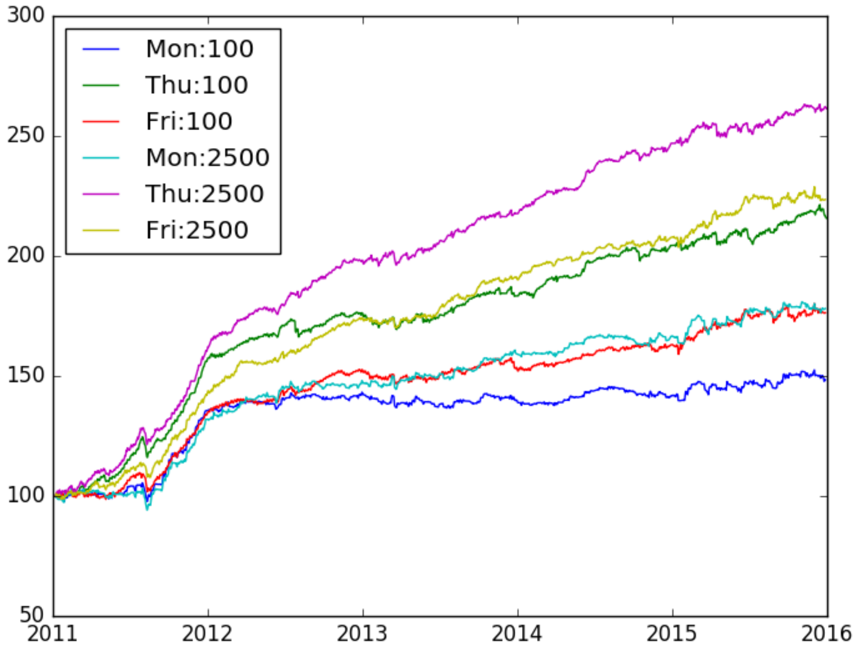

Figure 3: Track-record of the back-tested performance of the delta-hedging applied to weekly straddles. Legend labels stand for day of week on which the delta-hedged options were systematically sold and *threshold* parameter. Vertical axis corresponds to accumulated value (in percent) and horizontal axis corresponds to timeline.

Table 9: Average performance of the proposed delta-hedging algorithmic strategy in terms of vega risk (1.00 is equivalent to net pnl equal to 1.00 vega risk of the sold option). To remind, average bid-offer for weekly at-the-money EURUSD straddles is typically 0.70-wide in terms of implied volatilities (e.g. 10.00/10.70). Back-testing assumed that options were sold at bid that was 0.40 lower than the market mid (e.g. 9.95 assuming 10.35 as mid-market). Only data for options from **subset1** is shown.

threshold	Mon	Tue	Wed	Thu	Fri
			2011		
100	1.16	0.80	0.83	1.62	0.99
500	1.29	0.95	1.05	1.86	1.15
1000	1.37	0.91	1.17	1.80	1.29
2500	1.04	1.00	1.17	1.77	1.21
5000	1.04	1.10	0.97	1.83	1.31
7500	0.92	1.11	1.05	1.68	1.14
10000	0.80	1.19	1.02	2.07	1.04
			2012		
100	0.21	0.19	0.38	0.50	0.47
500	0.38	0.36	0.65	0.86	0.68
1000	0.54	0.53	0.67	0.86	0.77
2500	0.47	0.56	0.55	0.97	0.85
5000	0.79	0.46	0.34	0.92	0.72
7500	0.15	0.37	0.45	1.03	0.51
10000	0.52	0.27	0.38	0.89	0.67
			2013		
100	-0.10	0.17	0.10	0.25	0.00
500	0.19	0.30	0.20	0.43	0.23
1000	0.35	0.33	0.33	0.44	0.28
2500	0.37	0.39	0.23	0.61	0.47
5000	0.10	0.34	0.18	0.38	0.32
7500	0.41	0.19	0.12	0.62	0.23
10000	0.38	-0.01	0.21	0.39	0.39
			2014		
100	0.10	0.04	0.17	0.61	0.31
500	0.24	0.19	0.23	0.78	0.29
1000	0.18	0.14	0.24	0.69	0.36
2500	0.24	0.21	0.16	0.84	0.50
5000	0.20	0.24	0.18	0.97	0.31
7500	0.19	0.27	0.27	0.98	0.54
10000	0.19	0.19	0.48	0.74	0.58
			2015		
100	0.18	-0.28	-0.21	0.32	0.40
500	0.17	-0.07	-0.10	0.46	0.39
1000	0.31	0.04	-0.02	0.43	0.46
2500	0.34	0.10	0.00	0.38	0.44
5000	0.01	0.08	-0.18	0.58	0.74
7500	0.16	0.22	-0.16	0.38	0.53
10000	0.08	0.05	-0.09	0.08	0.46
			average		
100	0.31	0.18	0.25	0.66	0.43
500	0.45	0.35	0.41	0.88	0.55
1000	0.55	0.39	0.48	0.84	0.63
2500	0.49	0.45	0.42	0.91	0.69
5000	0.43	0.44	0.30	0.94	0.68
7500	0.37	0.43	0.35	0.94	0.59
10000	0.39	0.34	0.40	0.83	0.63